A SOLUTION OF SHAPE EXTRUSION OR DRAWING BASED ON DEFORMATION THEORY①

Yuan Xi
Department of Materials Science and Engineering, Central South University of Technology, Changsha 410083

ABSTRACT A new method based on deformation theory for analysing shape extrusion or drawing was described. The processes of the extrusion or drawing of regular polygonal or rectangular sections from cylindrical billets were discussed. The calculated results have been compared with known data obtained from upper bound method and were found to be in good agreement.

Key words shape extrusion or drawing deformation theory upper bound approximation ideal deformation

1 INTRODUCTION

Since stress-total strain relations were introduced by Hencky for a von Mises material in 1924, a great deal of discussion has followed on the question of whether the deformation theory could adequately represent an elastic-plastic material. While it is now evident that it cannot, due to its comparative simplicity as opposed to the incremental theory and its capability to approximate, rather than replace incremental theory, the deformation theory has gained a number of application and yielded some good results. In the 1970s', Martin[1] pointed out that the stress-total strain relations are appropriate to represent an elastic-plastic material if and only if a deformation is along the so called 'extremal path', thus gave the deformation theory a theoretical basis. In the field of metal forming, the theory was mostly used to analyse sheet metal forming processes[2-5], the tube sinking[6], cylinder and ring upsetting processes[7], and cold Pilgering processes[8] were also analysed. The results of those works show the viability of the deformation theory on analysing the large deformation processes of metal forming.

As to the complicated three-dimensional deformation processes like the shape extrusion or drawing, upper bound method has been used by the majority of researchers, such as Kiuchi *et al*[9-11], Yang *et al*[12, 13], Gunasekera *et al*[14, 15] in the past two decades. Different kinematically admissible velocity fields were assumed and some processes of simply-shaped sections were calculated. But this technique needs a complicated process of setting up a kinematically adimissible velocity field which makes it quite difficult to analyse working processes of complicated sectional products. FEM has also been used to analyse the similar problems more and more now, but its need of computation resource is much greater. Those drawbacks now become a major discouragement for the engineers who may want to use these approaches as a practical design tool. Thus a simple method is in much desire.

From the above reason, a new method based on deformation theory for analysing the 3-D deformation of shape extrusion or drawing is developed in this paper. By neglecting the strain history effect, the analysis can be greatly simplified. Through the analysis of a specific problem, the extrusion or drawing of regular polygonal or rectangular sections, this paper describes and discusses this new method

① Received Jan. 13, 1995; accepted Jun. 30, 1995

and gives it a theoretical basis.

2 ANALYTIC EXPRESSION OF EXTRUSION OR DRAWING OF REGULAR POLYGONAL OR RECTANGLAR SECTIONS FROM CYLINDRICAL BILLETS

As indicated by the deformation theory, when the geometries of a deforming body before and after the deformation are given, the strain field as well as the deformation work of the deformation process is solely specified. So by considering the geometries of the billet and the product of an extrusion or drawing process, all the information about the deformation can be obtained. A specific problem, the extrusion or drawing of regular polygonal or rectangular sections is discussed as follows. Here the deforming material is considered incompressible and non-strain-hardening and its velocities along the cross sections at the exit and the entrance of the deformation zone are assumed constant.

As shown in Fig. 1, OAB is half section of any of the polygon's sides and OA_0B_0 is the corresponding section of the cylindrical billet with its radius of R, i. e. the section OA_0B_0 move onto section OAB after deformation preserving the extrusion (or drawing) ratio. If point (X, Y, Z) on section OA_0B_0 move onto (x, y, z) on section OAB after deformation, suppose a constant deformation ratio along the

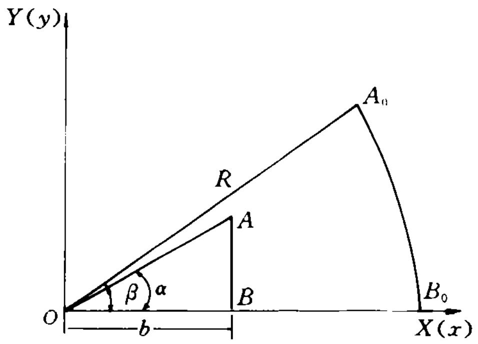

Fig. 1 Geometry of billet and produt

exit cross section, the displacement field of the process which meets the needs of the above assumptions can be easily gained as:

$$\left.\begin{aligned} x &= b\sqrt{(X^2+Y^2)}/R \\ y &= \frac{Kb}{R}\sqrt{(X^2+Y^2)}\arctan\frac{Y}{X} \\ z &= \lambda Z \end{aligned}\right\} \tag{1}$$

where

$$\left.\begin{aligned} \lambda &= \frac{\beta R^2}{b^2\tan\alpha} \\ K &= \tan\alpha/\beta \end{aligned}\right\} \tag{2}$$

The components of the right Cauchy-Green tensor $\boldsymbol{C}$ of this field are

$$\begin{aligned} &C_{11}=K'[X^2+K^2(X\arctan(Y/X)-Y)^2] \\ &C_{22}=K'[Y^2+K^2(Y\arctan(Y/X)+X)^2] \\ &C_{33}=\lambda^2 \\ &C_{12}=C_{21}=K'[XY+K^2(X\arctan(Y/X) \\ &\qquad -Y)(Y\arctan(Y/X)+X)] \\ &C_{13}=C_{31}=C_{23}=C_{32}=0 \end{aligned} \tag{3}$$

where

$$K' = b^2/[R^2(X^2+Y^2)]$$

The principle values of tensor $\boldsymbol{C}$ are

$$\mu_{1,2} = \frac{b^2}{2R^2}\{1+K^2(1+\arctan^2\frac{Y}{X}) \pm\sqrt{[(1+K^2(1+\arctan^2\frac{Y}{X}))^2-4K^2]}\}$$

$$\mu_3 = \lambda^2 \tag{4}$$

As proposed by Hill[16], the logarithmic strain $E=\ln C/2$ is choosen as the measure of the finite strain of the deformed body.

$$\begin{aligned} E_{1,2} &= \ln\mu_{1,2}/2 \\ E_3 &= \ln\lambda \end{aligned} \tag{5}$$

The principle direction of the strain is the same as that of the right Cauchy-Green tensor's, i. e. one principle axis is parallel with Z-axis and the direction cosine of the other two, l, m, are

$$\left.\begin{aligned} l &= \frac{C_{12}}{\sqrt{[(C_{11}-\mu)^2+C_{12}^2]}} \\ m &= \frac{C_{11}-\mu}{\sqrt{[(C_{11}-\mu)^2+C_{12}^2]}} \end{aligned}\right\} \tag{6}$$

It can be easily proved that $\mu_1\mu_2\mu_3=1$, so the law of incompressibility is satisfied.

The effective strain is

$$E_e = \sqrt{[2(E_1^2+E_2^2+E_3^2)/3]} \tag{7}$$

The work consumed per unit time of any

point "i" is

$$W(E_i^s) = \sigma_s E_{ei} \qquad (8)$$

where σ_s is average yield stress of the process.

3 DISCUSSION

(1) The exact solution for the problem, obtained incrementally, is characterized by displacement u_i^t, strain E_i^t and consumed work $W(E_i^t)$. The solution obtained by above approach is represented as u_i^s, E_i^s, $W(E_i^s)$.

Because of the fact that E_i^s is obtained from an extremal path, E_i^s and thus $W(E_i^s)$ are equal to or less than the integrated deformation strain and work obtained from incremental measurement of the real process which produces the same change in shape[1]. Besides, the supposed displacement u_i^s takes no consideration of displacemnt difference of adjacent points along Z-direction and thus takes no consideration of shear work along Z-direction, so

$$W(E_i^s) \leqslant W(E_i^t) \qquad (9)$$

i. e. $W(E_i^s)$ is the lower bound of the true deformation work.

Generally, the deformation whose consumed work hasn't any redundant work is called ideal deformation. It has been proved that the deformation theory's solution of any given elastic-plastic problems has the minimum potential work [1], so the ideal deformation can be strictly defined as that obtained by the deformation theory. This indication confirms the Majiles *et al*'s demonstration[3] that their approach based on deformation theory is specially useful to metal forming processes that occur under an idealized homogeneous mode of deformation. Similarly, the approach described above would also give its results specially accurate in the analysis of an idealized homogeneous deformation and the strain calculated in eqaution (7), the work calculated in equation (8) can be approximately called ideal strain and ideal work respectively.

(2) As indicated above, $W(E_i^s)$, the internal power of a deformation, is accounted for only by the ideal portion. To make a contrast, its total deformation work $\underset{\sim}{W}_\Sigma(E^s)$ obtained from the upper bound method is the sum of the deformation work $\underset{\sim}{W}_\Sigma(E_i^s)$ inside the deformed body and the shear work $W_{sh}(E_i^s)$ along the velocity discontinuity surfaces near the inlet and outlet of the extrusion or drawing dies, i. e.

$$\underset{\sim}{W}_\Sigma(E_i^s) = \underset{\sim}{W}(E_i^s) + W_{sh}(E_i^s) \qquad (10)$$

Considering $W(E_i^s)$ is the consumed work along extremal path while $\underset{\sim}{W}(E_i^s)$ is of a kinematically adimissible velocity field, thus

$$\underset{\sim}{W}(E_i^s) \leqslant \underset{\sim}{W}(E_i^s) \qquad (11)$$

So $W(E_i^s)$ gives a too low result compared with $\underset{\sim}{W}_\Sigma(E_i^s)$ which strongly supports the addition of the shear or redundant power along surfaces of velocity discontinuity, i. e.

$$W_\Sigma(E_i^s) = W(E_i^s) + W_{sh}(E_i^s) \qquad (12)$$

Similar procedure was once adopted as a modification of equilibrium approach for wire drawing[17, 18] and showed more of an upper bound nature[19]. The derivation of $W_{sh}(E_i^s)$ in this paper also follows that by them[17, 18].

4 RESULTS

The calculated results and some corresponding results given by published upper are showed in Fig. 2 and Fig. 3, where the avarage effective strain and relative stress is defined as

$$\overline{E}_e = \frac{\int_F W_\Sigma(E_i^s)\mathrm{d}F}{\int_F \sigma_s \mathrm{d}F} = \overline{P}/\sigma_s \qquad (13)$$

where F is the cross sectinal area of products.

Yang *et al*[12] proposed a kinematically admissible velocity field with smooth transition from the die entrance to the die exit. Due to their adoption of effective die surface with small friction, the deformation obtained by them, both theoretically and experimentally, was quite homogeneous, especially when the extrusion ratio is small. Fig. 2 compares the results obtained in this paper (curve 1, 2) with theirs. It can be seen from the figure that, just as predicated, the curve 1 which is calculated by substituting equation (8) into equation (13) has the smallest value and is quite similar to that obtained by Yang et al especially when the reduction of area is small. Besides the curve 4 which is calculated by substituting equation (12) into equation (13) has higher value than that of curve 3, which shows it is an upper bound of the true solution already.

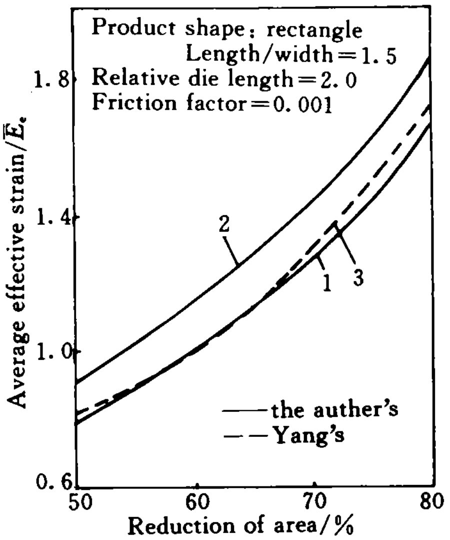

Fig. 2 Comparision with Yang for the extrusion of rectangle

1—Results without taking account of redundant work; 2—Results having taken account of redundant work

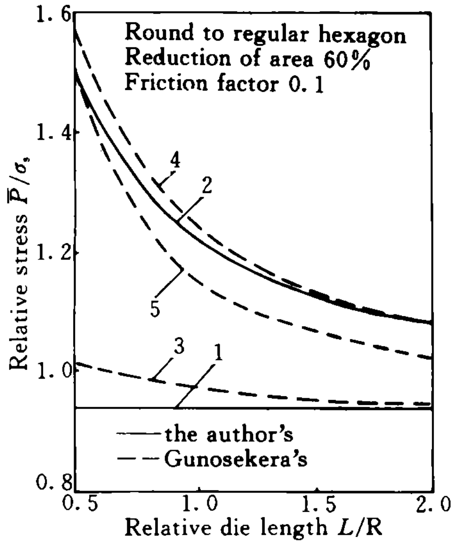

Fig. 3 Comparision with Gunasekera for the extrusion of regular hexagon

1, 3—Results without taking account of redundant work; 2, 4, 5—Results having taken account of whole deformation work

Fig. 3 shows the results obtained by substituting equation (8) and (12) into equation (13) (curve 1, 2) and the results provided by Gunasekera *et al*[14, 15] who adopted straight flow line supposition (curve 3, 4) and the smooth flow line supposition (curve 5). The relative die length L/R in the figure is the ratio of deformation zone length L and the billet's radius R. It can be observed that curve 1 has the smallest value and curve 2 is higher than curve 5, which share the same regularity with Fig. 2. Furthermore, when L/R goes greater and greater, curve 2 approachs its mininum relative stress value and becomes almost the same as curve 4.

5 CONCLUSIONS

Based on the deformation theory, the distribution of strain and the deformation work of the extrusion or drawing of regular polygonal or rectangular sections from cylindrical billets are explicitly formulated from a supposed displacement field in this paper. The substructure of the solution is discussed and its characteristic of lower bound and the physical meaning are pointed out. Furthermore, a modified method is developed to modify the above lower bound solution to an upper bound solution. The calculated results show their similarity to homogeneous deformation and upper bound solutions just as predicted.

The present method shows its preponderence of simplicity in analysing the working process of complicated sections, which may be make it possible for engineers to use it as a computerized design tool. Extending the method presented in this paper to other generally shaped sections is easy and the further work will be published soon.

REFERENCES

1 Martin J B. Plasticity:Fundmantals and General

(*To page 126*)

5 CONCLUSIONS

(1) There was not interface phase in the specimen after quenching and subsequently aging at below 800 ℃. This means that the forming of the interface phase is related to heating temperautre and cooling rate.

(2) There was interface phase in the specimen after treating at 950 ℃ for 1 h and water-quenching followed by aging at 800 ℃. The interface phase had a double-layer structure with it. The monolithic layer was *fcc* and its orientation is $(001)_m // (110)_\beta$ and $[110]_m // [1\bar{1}\bar{1}]_\beta$. The striated one was *HCP* with $(10\bar{1}1)\langle\bar{1}012\rangle$ twin relationship to α phase. Its thickness was about 100～300 nm.

(3) When the interface phase formed, the composition gradient at the interface occurred, which resulted in gradient of electron concentration. This probably is the main factor to create the interface phase in the titanium alloys. If no interface phase was formed, the concentration of alloying elements was discontinues at α/β boundaries.

(4) The interface phase of titanium alloys has nothing to do with the thinning method for TEM specimens and the so-called titanium hydride.

REFERENCES

1 Rhodes C G, Williams J C. Metall Trans, 1975, 6A: 1670.
2 Moody N R, Creulich F A, Robinson S L. Metall Trans, 1984, 15A: 1955.
3 Rhodes C G, Paton N E. Metall Trans, 1979, 10A: 1753.
4 Banerjee D, Arunachalam V S. Acta Metall, 1981, 29: 1685.
5 Rhodes C G, William J C. Metall Trans, 1975, 6A: 2103.
6 Banerjee D. Metall Trans, 1982, 13A: 681.
7 Banerjee D, Shellon C G *et al.* Acta Metall, 1988, 36: 125.
8 Rhodes C G, Paton N E. Metall Trans, 1979, 10A: 209.
9 Margolin H, Levine E, Young M. Metall Trans, 1977, 8A: 373.
10 Mahajan Y, Nadiv S, Fujidhiro. Scripta Metall, 1982, 16: 375.
11 Fisher E S, Dever D. Acta Metall, 1970, 18: 265.

(Edited by He Xuefeng)

(From page 117)

Results. Massachusetts: MIT Press, 1975.
2 Гун Г Я *et al.* Пластическое формоиз менение металлов. Moscow: Металлургия, 1968. 416.
3 Storen S, Rice J. J Mech Phys Solids, 1975, 23 (3): 421—441.
4 Majiles S A, Lee D. J Eng Ind, 1987, 109(4): 330—337.
5 Majiles S A, Lee D. J Eng Ind, 1993, 115(1): 102—109.
6 Avitzur B. Metal Forming. Proceses and Analysis. New York: McGraw-Hill, 1968. 341—351.
7 Vertin K D, Majiles S A. J Eng Ind, 1993, 115 (3): 450—458.
8 Furugen M, Hayashi C. J Mech Work Technol, 1984, 10(3): 273—286.
9 木内学，岸英敏，石川政和. 塑性と加工，1983，24(3): 290—296.
10 木内学，石川政和. 塑性と加工，1983，24(7): 722—729.
11 木内学，石川政和. 塑性と加工，1984，25(7): 604—611.
12 Yang D Y, Lee C H. Int J Mech Sci, 1978, 20 (8): 541—552.
13 Yang D Y, Lee C H, Kim M U. Int J Mech Sci, 1986, 28(8): 517—534.
14 Gunasekera J S, Hoshino S. J Eng Ind, 1982, 104(1): 38—45.
15 Gunasekera J S, Hoshino S. J Eng Ind, 1985, 107(3): 229—233.
16 Hill R J. Mech Phys Solids, 1968, 16(4): 315—322.
17 Korber G, Eichinger A. Stahl u. Eisen, 1940, 60: 829—832, 854—881.
18 Siebel V E. Stahl u. Eisen, 1947, 67: 171—180.
19 Mot E. Trans ASME: J Eng Ind, 1968, B90: 742—744.

(Edited by Zhu Zhongguo)